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OCTONIONIC PLANES AND REAL FORMS OF G_2 , F_4 AND E_6

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In this work we present a useful way to introduce the octonionic projective and hyperbolic plane $\mathbb{O}P^2$ through the use of Veronese vectors. Then we focus on their relation with the exceptional Jordan algebra $\mathfrak{J}_3^{\mathbb{O}}$ and show that the Veronese vectors are the rank-one elements of the algebra. We then study groups of motions over the octonionic plane recovering all real forms of G₂, F₄ and E₆ groups and finally give a classification of all octonionic and split-octonionic planes as symmetric spaces.

MSC: 17C36, 17C60, 17C90, 22E15, 32M15 *Keywords*: Exceptional Lie groups, Jordan algebra, octonionic projective plane, real forms, Veronese embedding

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1. Introduction

The study of the exceptional Jordan algebra and its complexification has been of interest in recent papers of theoretical physics. Todorov, Dubois-Violette [18] and doi: 10.7546/giq-23-2022-39-57 39