



NUMERICAL RANGES OF THE REAL 2×2 MATRICES DERIVED BY FIRST PRINCIPLES

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Here we demonstrate how the very definition of the numerical range leads to its direct geometrical identification. The procedure which we follow can be even slightly refined by making use of the famous Jacobi's method for diagonalization in reverse direction. From mathematical point of view, the Jacobi's idea here is used to reduce the number of the independent parameters from three to two which simplifies significantly the problem. As a surplus we have found an explicit recipe how to associate a Cassinian oval with the numerical range of any real 2×2 matrix. Last, but not least, we have derived their explicit parameterizations.

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