GIQ **26** (2023) 39–52

SERIES ON
Geometry, Integrability and Quantization
ISSN 1314-3247

ON THE SPECTRUM OF THE DISCRETE BILAPLACIAN WITH RANK-ONE PERTURBATION

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Communicated by Andreas Arvanitoyeorgos

We study the spectral properties of the family of self-adjoint bounded discrete Schrödinger-type operators in the Hilbert space of square-summable complex-valued functions defined on the d-dimensional lattice. We completely describe the discrete spectrum of Schrödinger-type operators lying outside the essential spectrum and study the location and the uniqueness, analyticity, monotonicity and convexity properties of eigenvalues depending on all of the parameters. Moreover, in the case of existence we obtain the asymptotics of eigenvalues of Schrödinger-type operator.

MSC: 81Q10, 47A10, 47A55, 47A75, 47J10
Keywords: Asymptotics, discrete bilaplacian, discrete spectrum, eigenvalues, essential spectrum, expansion

1. Introduction

In this paper we investigate the spectral properties of the perturbed discrete biharmonic operator

$$\widehat{\mathbf{H}}_\mu := \widehat{\mathbf{H}}_0 - \mu\widehat{\mathbf{V}}, \qquad \mu \geq 0 \tag{1}$$

of self-adjoint bounded discrete Schrödinger-type operators in the Hilbert space $\ell^2(\mathbb{Z}^d \times \mathbb{Z}^d)$ of square-summable complex-valued functions on cartesian product $\mathbb{Z}^d \times \mathbb{Z}^d$ and $\mathbb{Z}^d$ is the d-dimensional cubical lattice. Here $\widehat{\mathbf{H}}_0$ is discrete bilaplacian, i.e.,

$$\widehat{\mathbf{H}}_0 = \widehat{\Delta}\widehat{\Delta} \otimes \widehat{I} + \widehat{I} \otimes \widehat{\Delta}\widehat{\Delta}$$

where

$$\widehat{\Delta} f(x) = \frac{1}{2}\sum_{|s|=1}(f(x) - f(x+s)), \qquad f \in \ell^2(\mathbb{Z}^d \times \mathbb{Z}^d)$$

doi: 10.7546/giq-26-2023-39-52