



# RICCI CURVATURE OF CAYLEY GRAPHS FOR DIHEDRAL, GENERALIZED QUATERNION AND CYCLIC GROUPS

IWAO MIZUKAI and AKIFUMI SAKO

Presented by Akifumi Sako

Lin, Lu, and Yau formulated the Ricci curvature of edges in simple undirected graphs. Using their formulation, we calculate the Ricci curvatures of Cayley graphs for the dihedral groups, the general quaternion groups, and the cyclic groups with some generating sets chosen so that their cardinal numbers are less than or equal to four. For the dihedral group and the general quaternion group, we obtained the Ricci curvatures of all edges of the Cayley graph with generator sets consisting of the four elements that are the two generators defining each group and their inverse elements. We deal with Cayley graphs where two different edge directions are determined for each vertex. The introduction of two types of edges is intended to discretize the fiber bundle, separating the edges in the horizontal and vertical directions.

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## Contents

<b>1</b>	<b>Introduction</b>	<b>52</b>
<b>2</b>	<b>Preparations</b>	<b>56</b>
2.1	Ricci Curvature of Graphs . . . . .	56
2.2	Cayley Graphs . . . . .	57
<b>3</b>	<b>Ricci Curvature of the Cayley Graphs of Dihedral Groups</b>	<b>58</b>
<b>4</b>	<b>Ricci Curvature of the Cayley Graphs for Generalized Quaternion Groups</b>	<b>63</b>
4.1	Generalized Quaternion Groups . . . . .	63
<b>5</b>	<b>Ricci Curvature of the Cayley Graphs for Cyclic Groups</b>	<b>65</b>
5.1	Cyclic groups and Cayley graphs . . . . .	65
5.2	Ricci Curvature of the Cayley Graph for Cyclic Groups $\Gamma(\mathbb{Z}/n\mathbb{Z}, S_1)$ with a Generating Set $S_1 = \{+1, -1\}$ . . . . .	65
doi: 10.7546/giq-28-2024-51-79		51