



ASPECTS OF NONCOMMUTATIVE DYNAMICS: PARTICLES

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Dynamics on noncommutative spaces has long been expected to solve the problem of infinities in quantum field theory. While this hope remains unconfirmed to date, surprising effects emerged, testing some of the limits of present theories. Noncommutative theories enlarge by necessity fundamental concepts like gauge invariance, (non)locality, causality, or dimensional reduction. This makes them an excellent theoretical laboratory, in which fundamental geometric concepts and quantization can be extended in a new but controllable and often quite suggestive set-up, situated “just beyond” the current paradigm. On the more practical side, noncommutativity offers some unexpected technical solutions, for instance it provides a natural discretization of space – which takes care of *classical* field theory divergences – and a simple way to put fermions on a lattice. Noncommutativity appears in two different forms, called here “particle noncommutativity” and “field noncommutativity”. This work reviews the case of “particle noncommutativity” and then shows how fields are associated with nonconstant commutators.

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