



FUNDAMENTAL EQUATIONS OF GENERALIZED $\varphi(\text{Ric})$ -VECTOR FIELDS

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The paper treats pseudo-Riemannian spaces admitting generalized $\varphi(\text{Ric})$ -vector fields. We investigate the fundamental equations governing the existence of such vector fields, which are expressed as closed linear partial equations of the Cauchy type. In particular, we examine these equations within the context of constant curvature and Kähler spaces.

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1. Introduction

Let \mathbb{V}_n denote an n -dimensional pseudo-Riemannian space equipped with a metric tensor g_{ij} . Hinterleitner and Kiosak in [4, 5] introduced a concept of $\varphi(\text{Ric})$ -vector fields which are vector fields obeying to the equations $\varphi_{i,j} = cR_{ij}$, where R_{ij} is the Ricci tensor, c is some constant, and comma “,” denotes covariant derivative with respect to the connection on \mathbb{V}_n .

In their work, the aforementioned authors explored several geometric properties of these vector fields. Specifically, they examined conformally flat pseudo-Riemannian spaces and spaces that simultaneously admit $\varphi(\text{Ric})$ -vector fields as well as concircular vector fields.

The authors of [8] generalized these vector fields, referring to them as *generalized $\varphi(\text{Ric})$ -vector fields* in the following way

$$\varphi_{i,j} = sR_{ij} \tag{1}$$

where s is a certain function on \mathbb{V}_n .

These vector fields exist in Kagan subprojective spaces. In [8], the authors established certain properties of these vector fields within conformally Euclidean and Kähler spaces.