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QUANTIZATION OF CONSTRAINT SYSTEM AND HOMOLOGICAL COMMUTATION RELATION OF ELECTRIC AND MAGNETIC FLUXES

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We prove that a commutation relation of the electric flux and the magnetic flux operators is proportional to the crossing number of the surfaces defining the fluxes. To prove it we use Dirac's prescription for quantization of constraint system and de Rham's current forms which are geometric generalizations of so-called delta function.

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1. Introduction

Quantum field theory typically focuses on the commutation relations of pointwise local field operators. However, this approach often obscures the global properties of observable operators. In a previous study [7], we demonstrated that electric and magnetic fluxes, defined through surface integrals of local field operators, satisfy a canonical commutation relation. In this study, we extend that result by incorporating more general integrals of local field operators, utilizing the concept of currents from differential geometry.

2. Main Results

One of the main results of this study is the commutation relation

$$\Big[\int_{\underline{M}} (\hat{E} \wedge \beta), \int_{\underline{M}} (\hat{A} \wedge \gamma) \Big] = \mathrm{i}\hbar \hat{1} \Big(\int_{\underline{M}} (\mathrm{d}^\dagger \beta_+ \wedge \mathrm{d}\gamma_-) + \int_{\underline{M}} (\beta_0 \wedge \gamma_0) \Big) \qquad (1)$$

for quantum-theoretical operators of the electric field $\hat{E}$ and of the vector potential $\hat{A}$. In our notation, $\hat{E}$ is a twisted two-form and $\hat{A}$ is a one-form. β is an arbitrary one-form and $\beta = \mathrm{d}\beta_- + \mathrm{d}^\dagger \beta_+ + \beta_0$ is its Hodge decomposition. γ is an arbitrary