



# BLOCH-MESSIAH DECOMPOSITIONS OF SYMMETRIC RAY TRANSFER MATRICES IN $\mathbb{R}^4$

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Here we present a plethora of explicitly parameterized real symmetric symplectic matrices in dimension four. It turns out that structurally these matrices depend on a set of six real parameters and a constructive formula for building them by these parameters is derived.

On the other side, assuming the possibility of Bloch-Messiah decomposition, makes clear the fact that the set of these matrices splits naturally as a product of rotational and squeezing transformations.

This possibility is proved by presenting an algorithmic procedure for performing the factorization of an arbitrary real symmetric symplectic matrix and this is illustrated via numerous explicit examples.

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