GIQ **31** (2025) 67–69

SERIES ON
Geometry, Integrability and Quantization
ISSN 1314-3247

BOOK REVIEW

Orthogonal Curvilinear Coordinates [*and more*] *for Engineers and Physical Scientists*, Tomás Soler and Jen-Yu Han, Chicago 2024, x + 560 pages

The challenges facing modern mathematical-physics should be open to the scrutiny of new tools and ideas. For example, Terence Tao made this clear in his article [6] when he tried to solve the three-dimensional Navier-Stokes equation. However, in order to adopt novel mathematical concepts, it is necessary to master first well-known fundamentals. Among them, why not, is the prospect of searching into the classical theory of orthogonal curvilinear coordinates, which is the subject of the book by Thomás Soler and Jen-Yu Han, explicitly titled "Orthogonal Curvilinear Coordinates [and more] for Engineers and Physical Scientists".

This new publication will be very useful for advanced students of physical and engineering specialties, as well as curious graduates planning to engage in scientific activities, since only limited number of physical problems can be solved in rectangular Cartesian coordinates. Quite often, one has to switch to their cylindrical and/or spherical counterparts, and therefore, it is necessary to work from the start with the prevalent Jacobians matrices. It is crucial to have reliable reference sources in your library, since materials that can be found on the Internet often contain careless errors. I remember how, when developing a new mathematical technique for modeling turbulent and laminar boundary layers, in order not to make a mistake, I manually performed all the calculations for the transition of Navier-Stokes equations from rectangular Cartesian coordinates to cylindrical and spherical ones [4, 5]. And this is quite tedious work because, for example, Laplacians of velocity field components in spherical coordinate system have at least five terms (almost all of them with variable coefficients)! Therefore, having in your own personal library comprehensive reference alternative guide such as the recent book by Soler and Han will, undoubtedly, facilitates future scientific inquiry.

The authors use matrix notations from the very beginning even for classical vectors operations, which may seem a bit unnecessary, but later this approach allows them to logically expand it to much more complex themes, such as differential matrix operators, introducing in the process so-called matrix-tensors of third-rank and defining innovative symmetries by blocks, columns and rows to understand their

doi: 10.7546/giq-31-2025-67-69

relationships. Personally, I first learned that each vector can be assigned an antisymmetric tensor from a textbook on theoretical physics [3]. At the same time, the paperback under review starts right away with this basic concept (the term skew-symmetric matrix is used here). Thus, by studying this volume, one can obtain adequate alternate pathways for analyzing the specialized physics literature. By evaluating the chapters highlighting the role of Jacobians protagonism, students can prepare themselves for tackling thermodynamics, where it is often necessary to translate differential equations from one set of physical parameters to another. The amount of detail led by an ingenious approach focused on matrix notations and calculus offered by this book allows one to forge better understanding of continuum mechanics, where standard differential operators are used in a similar way but without invoking matrices. And, of course, the narrative could be useful to students learning theoretical mechanics [3], where various kinds of tensors (for example, the inertia tensor) and coordinates (such as elliptic coordinates) are an integral part of the syllabus.

The book consists of six chapters. Each chapter contains material that is especially valuable from a physicist's point of view. Thus, the first chapter presents different types of vector products (1.5) (in addition to scalar and vector, there are dyadic, tensor and other products). Chapter 2 is devoted to working with Jacobians, metric, Lamé and rotation matrices; it presents formulas for transformations between curvilinear and Cartesian coordinates (2.3). Particularly useful for physicists will be the third chapter, which describes differential matrix operators, since virtually every equation in physics contains gradients, divergences, rotors and Laplacians from different fields. Also in this chapter, one can find a representation of the inertia tensor up to the fifth rank (3.10) and a fairly complete exposition of the theory of gravitational potential (3.12—3.16). The Christoffel symbols, which are presented in Chapter 4, will obviously be more useful to physicists who use numerical methods in their work, for analytical verification of certain results. The material of Chapter 5 will be especially useful in solving problems. Here one can find relations for time derivatives of unit vectors (5.7), expressions for velocity and acceleration in curvilinear coordinates (5.8), and other important data. Miscellaneous applications are given in Chapter 6. Students of elasticity theory will find the representations of the strain and stress tensors (6.1—6.14) in curvilinear coordinates useful. Since I am currently modeling turbulent and laminar boundary layers, the material devoted to the Navier-Stokes equation (6.15 and 6.16) was of particular interest to me. I carefully checked the equation in spherical coordinates presented in the book with similar results that I obtained manually, and did not find a single typo (while the material that can be found on the Internet almost always contains typos).

In terms of the scope of the material, Soler and Han's book can be compared to McConnell's book on tensor analysis [2], although the latter has only one chapter devoted to curvilinear coordinates. If we compare this book with those that are widespread in universities in the territory of the former Soviet Union, then here we can highlight the book by Borisenko and Tarapov [1], devoted to vector and tensor analysis. This book contains almost half as many pages, and the material devoted to physical applications is printed in small print. Therefore, the book by Borisenko and Tarapov is quite compact and can be easily carried around, which, of course, cannot be said about the book by Soler and Han: to carry this book around you need to have a fairly large muscle mass. However, the advantage of the latter is a more fundamental presentation of the material and the absence of typos. The main drawback of Soler and Han's book, from the point of view of a physicist, is that their content does not include a section in the Appendix discussing the general matrix formulation to address problems in cylindrical coordinates, which physicists have to deal with very often. In conclusion, this is a new book that, in the opinion of the reviewer, could provide other scientists the opportunity to expand the theory of orthogonal curvilinear coordinates by exploiting further the skills of rigorous matrix handling. Time only knows.

References

[1] Borisenko A. and Tarapov I., *Vector Analysis and the Beginnings of Tensor Calculus* (in Russian), Vyshcha Shkola, Kharkiv 1986.

[2] McConnell A., *Tensor Analysis*, Dover, New York 1957.

[3] Landau L. and Lifshitz E., *Mechanics*, Butterworth-Heinenann, Oxford 1960.

[4] Shvydkyi O., *Analytical Solution of the Steady Navier-Stokes Equation for an Incompressible Fluid Entrained by a Rotating Disk of Finite Radius in the Area of Boundary Layer*, Ann. Math. Phys. **7** (2024) 305–313.

[5] Shvydkyi O., *Sharp Cone-Broad Cone-Disk*: *Analytical Solutions in the Tunnel Mathematics Space to the Steady Navier-Stokes Equations in the Area of Boundary Layer for Incompressible Symmetric Flows Entrained by These Rotating Bodies*, Acceleron Aerospace J. **3** (2024) 624–647.

[6] Tao T., *Finite Time Blowup for an Averaged Three–Dimensional Navier–Stokes Equation*, arXiv:1402.0290v3 [math.AP].

Oleh G. Shvydkyi
Independent Researcher
Zaporizhzhia, UKRAINE