



CRITICAL CURVES AND THRESHOLD RESONANCES FOR A TWO-FERMION SCHRÖDINGER OPERATOR ON THE ONE-DIMENSIONAL LATTICE

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Communicated by Josef Mikeš

We investigate the Schrödinger operators describing a two-fermion system on the one-dimensional lattice with a fixed quasi-momentum and nearest- and next-nearest-neighbor interactions. In the two-parameter interaction plane, there exist two *critical* curves (hyperbolas) on which the Fredholm determinant vanishes, corresponding to the lower threshold of the essential spectrum. We provide a complete analytic description of the spectral behavior on these critical curves: on the first curve no bound states occur and the lower threshold is a *super-threshold resonance*, while on the second curve exactly one bound state exists below the essential spectrum and the lower threshold remains a super-threshold resonance.

MSC: 47A10, 47A55, 47A75, 47J10, 81Q10

Keywords: Critical curves, discrete Schrödinger operator, Fredholm determinant, lattice system, threshold resonance, two-fermion system

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