



SINGULARITIES, GEOMETRY AND SYMMETRIES OF DISCRETE DYNAMICAL SYSTEMS

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In this paper we give a review of integrability and geometry for completely integrable discrete finite dimensional dynamical systems. We rely mainly on the effect of singularities on the geometrical structure and symmetries. After describing the heuristic methods for singularity confinement and algebraic entropy we go to the rigorous formulation using the tools from algebraic geometry of rational elliptic surfaces. Using them we show how to construct the space of initial conditions by desingularisations and how invariants and symmetries appear naturally from the corresponding linear action on the associated Picard group. Four-dimensional discrete Painlevé equations are constructed by deautonomisations (allowing movable singularities) as well. All the results presented are based on joint work and private communications with Tomoyuki Takenawa and Anton Dzhamay.

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