



# STABILITY ANALYSIS OF EULER’S ELASTICA RING USING HARMONIC BALANCE

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This study presents a comprehensive stability analysis of elastic rings subjected to symmetric mode  $n$  loading. The nonlinear governing equation for the Euler elastica ring is solved using the harmonic balance method, which facilitates simplified yet accurate determination of stability ranges across various symmetry modes. The resulting bifurcation diagrams demonstrate that the harmonic balance method effectively predicts post-buckling behavior with smooth transitions, outperforming numerical Runge-Kutta solutions in capturing bifurcation structures. These findings establish a robust theoretical framework for elastic ring stability analysis relevant to diverse engineering applications.

MSC: 74B20, 74G60, 74K25, 74H55, 74S20

Keywords: Euler’s elastica ring, harmonic balance method, post-buckling, stability analysis

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