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VARIATIONAL SYMMETRIES AND LIE REDUCTION FOR FROBENIUS SYSTEMS OF EVEN RANK

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Abstract. Let $\mathcal{I}$ be Frobenius system of even rank. Consider a closed two-form $\Pi \in \mathcal{I} \wedge \mathcal{I}$ of maximal rank. A vector field X such that $\mathcal{L}_X \Pi = 0$ is called a symmetry of Π. We determine the relationship between the solvable Lie group of symmetries of Π and the rank of the reduced system obtained from $\mathcal{I}$ by Lie reduction. For an Euler–Lagrange system of ODE's with the corresponding Lagrangian L, Π can be taken to be the differential of the Poincaré–Cartan form η_L. A symmetry of $\Pi = \mathrm{d}\eta_L$ is a variational symmetry of the Lagrangian L. A proof of Noether's theorem for Frobenius systems of even rank is provided.

1. Introduction

This paper, like many others mentioned below, treats Lie symmetry method for systems of ordinary differential equations (ODE's) in the framework of exterior differential systems. Systems of differential equations in one independent variable may be interpreted as Pfaffian systems of codimension one. Such systems are examples of Frobenius (completely integrable) systems.

Lie's symmetry method, named after the famous Norwegian mathematician Sophus Lie, is one of the most successful methods for finding explicit solutions of systems of differential equations [10]. It unifies numerous methods used to integrate special types of equations, such as separable equations, homogeneous equations, Euler's equations, linear equations, Bernoulli equations, and many others. Lie's method has even more important applications into partial differential equations, but we will not discuss this aspect here. It was Sophus Lie who first realized that if one can associate a solvable s-parameter group