Fifth International Conference on
Geometry, Integrability and Quantization
June 5–12, 2003, Varna, Bulgaria
Ivaïlo M. Mladenov and Allen C. Hirshfeld, Editors
SOFTEX, Sofia 2004, pp 67–80

AN INTRODUCTION TO MOVING FRAMES

PETER J. OLVER

School of Mathematics, University of Minnesota
Minneapolis, MN 55455, USA

Abstract. This paper surveys the new, algorithmic theory of moving frames. Applications in geometry, computer vision, classical invariant theory, and numerical analysis are indicated.

1. Introduction

The method of moving frames ("repères mobiles") was forged by Élie Cartan, [7, 8], into a powerful and algorithmic tool for studying the geometric properties of submanifolds and their invariants under the action of a transformation group. However, Cartan's methods remained incompletely understood and the applications were exclusively concentrated in classical differential geometry, see [12, 13, 15]. In the late 1990's, we have formulated in [10, 11] a new approach to the moving frame theory that can be systematically applied to general transformation groups. The key idea is to define a moving frame as an equivariant map to the transformation group. All classical moving frames can be reinterpreted in this manner, but the new approach applies in far wider generality. Cartan's construction of the moving frame through the normalization process corresponds to the choice of a cross-section to the group orbits. Building on these two simple ideas, one may algorithmically construct moving frames and complete systems of invariants for completely general group actions. The existence of a moving frame requires freeness of the underlying group action.

Classically, non-free actions are made free by prolonging to jet space, leading to differential invariants and the solution to equivalence and symmetry problems via the differential invariant signature. More recently, the moving frame method was also applied to Cartesian product actions, leading to classification of joint invariants and joint differential invariants, [26]. The combination of jet and Cartesian product actions known as multi-space was proposed in [27] as a framework for the geometric analysis of numerical approximations, and, via the application of