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A NULL GEODESIC ORBIT SPACE WHOSE NULL ORBITS REQUIRE A REPARAMETRIZATION

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Abstract. We exhibit an example of a homogeneous Lorentzian manifold G/H whose homogeneous geodesics form just the light-cone and a hyperplane in the tangent space. For all light-like homogeneous geodesics, the natural parameter of the orbit is not the affine parameter of the geodesic.

1. Introduction

Homogeneous geodesics (see Definition 1) on homogeneous pseudo-Riemannian manifolds were studied both in physics [13, 14] and in mathematics [1–12, 15]. Penrose limits along null (that is, light-like) homogeneous geodesics are studied in [14]. It is shown there that the Penrose limit of a Lorentzian spacetime along a null homogeneous geodesic is a homogeneous plane wave and the Penrose limit of a reductive homogeneous spacetime along a null homogeneous geodesic is a reductive homogeneous plane wave. Null homogeneous geodesics and n.g.o. spaces (all null geodesics are homogeneous, see Definition 5) were introduced and studied in [13].

Riemannian and pseudo-Riemannian g.o. spaces (that is, spaces whose geodesics are all homogeneous) were studied in [5, 7, 8, 10] and the behaviour of geodesic graphs is investigated in [5]. Pseudo-Riemannian almost g.o. spaces (whose geodesics are almost all homogeneous) were studied in [3,6] and the behaviour of geodesic graphs in this case was investigated in [6]. The fundamental tool to determine homogeneous geodesics is the Geodesic Lemma (see Definition 1). Its generalization to the pseudo-Riemannian framework was proved in [4], and the existence of the two types of null homogeneous geodesics was illustrated in [1, 2, 4].