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THE GENERAL NOTION OF A CURVATURE IN CATASTROPHE THEORY TERMS*

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Abstract. We introduce a new notion of a curvature of a superconnection, different from the one obtained by a purely algebraic analogy with the curvature of a linear connection. The naturalness of this new notion of a curvature of a superconnection comes from the study of the singularities of smooth sections of vector bundles (Catastrophe Theory). We demonstrate that the classical examples of obstructions to a local equivalence: exterior differential for two-forms, Riemannian tensor, Weil tensor, curvature of a linear connection and Nijenhuis tensor can be treated in terms of some general approach. This approach, applied to the superconnection leads to a new notion of a curvature (proposed in the paper) of a superconnection.

1. A Brief Review of the Notion of a Superconnection

The notions of a superconnection and of the corresponding supercurvature were introduced by Quillen in 1985 [7]. In this section we give a brief review of the matter and introduce the basic notations.

By $\xi = (E, p, M)$ we denote a vector bundle over the manifold M ($\dim M = m$, $\dim(\xi) = n$), by ξ^* – the dual bundle and by $C^\infty(\xi)$ – the space of the vector fields, i.e., the space of the smooth sections of the bundle ξ. Respectively $\Omega^k(M) = C^\infty(\Lambda^k T^*(M))$ is the the space of the differential k-forms on the manifold M and

$$\Omega(M) = \bigoplus_{k=0}^{m} \Omega^k(M) = C^\infty\left(\bigoplus_{k=0}^{m} \Lambda^k T^*(M)\right)$$

*In memoriam of our dear friend and colleague Ventzeslav Rizov.