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PREQUANTIZATION OF SYMPLECTIC SUPERMANIFOLDS

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Abstract. This paper presents the formalism of symplectic supermanifolds with a non-homogeneous symplectic form and their prequantization.

1. Supermanifolds

The idea behind a supermanifold is that one wants to have *anticommuting* variables, i.e., a kind of "numbers" such that $\xi\eta = -\eta\xi$. In this context it is customary to denote ordinary/commuting (real) "numbers" by Latin characters and the anticommuting kind by Greek characters. One of the ideas to create such "numbers" is to replace the standard real line $\mathbb{R}$ by a graded commutative ring $\mathcal{A} = \mathcal{A}_0 \oplus \mathcal{A}_1$ and to take the commuting "numbers" x_i in the even part: $x_i \in \mathcal{A}_0$ and to take the anticommuting variety ξ_j in the odd part: $\xi_j \in \mathcal{A}_1$. The basic example of such a ring is the exterior algebra of an infinite dimensional (real) vector space E

$$\mathcal{A} = \bigwedge E = \Big(\bigoplus_{k=0}^{\infty} \bigwedge{}^{2k} E\Big) \oplus \Big(\bigoplus_{k=0}^{\infty} \bigwedge{}^{2k+1} E\Big) .$$

The first step in creating a theory of differential geometry based on these commuting and anticommuting "numbers," usually called even and odd coordinates, is to define what smooth functions are. When one tries to define the derivative of a function, one encounters immediately two problems: i) the most natural topology on the graded ring $\mathcal{A}$ is not Hausdorff making uniqueness of limits questionable and ii) due to nilpotent elements in $\mathcal{A}$ even a difference quotient is problematic. The solution adopted in [3] is based on the following two observations.

Lemma 1. *Let $U \subset \mathbb{R}^p$ be a convex open set and let $f : U \to \mathbb{R}^d$ be a function of class C^1. Then the function $g : U^2 \to \mathrm{End}(\mathbb{R}^p, \mathbb{R}^d) \cong \mathbb{R}^{pd}$ defined by*

$$g(x, y) = \int_0^1 f'\big(sx + (1-s)y\big)\, \mathrm{d}s$$