

Multicomponent Nonlinear Systems of Bose-Fermi Fields: Exact Solutions

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Abstract. We present families of stationary solutions for a multicomponent nonlinear system of two boson and N_f fermion fields in terms of elliptic functions of modulus k . This system is an extension of models, describing Bose-Fermi mixtures in the mean field approximation. We also single out the particular cases when the quasiperiodic solutions become periodic ones. In the limit of sinusoidal external potential ($k \rightarrow 0$) our solutions model periodic waves trapped in an optical lattice. The other limit $k \rightarrow 1$ provides solutions expressed by hyperbolic functions (vector solitons). Thus we demonstrate that our system describes quasi-periodic and periodic waves, as well as solitons.

Keywords: periodic solutions, solitons, nonlinear system of Bose-Fermi fields

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INTRODUCTION

Recently, there has been a strong interest on quantum degenerate mixtures of bosons and fermions [1]–[18]. Experimental realization of two-component Bose–Einstein condensates have stimulated considerable attention in general [17] and in particular in the quasi-1D regime [18, 19] when the Gross–Pitaevskii equations for two interacting Bose–Einstein condensates reduce to coupled nonlinear Schrödinger equations (CNLS) with an external potential. In specific cases the two component CNLS equations can be reduced to the Manakov system [20] with some external potential. These considerations increase the interest to study multicomponent nonlinear systems of Bose-Fermi fields closely related to CNLS, which have similar nature.

Important role in analyzing these effects was played by elliptic and trigonometric solutions of the above-mentioned equations. Such solutions for the one-component nonlinear Schrödinger equation are well known, see [21] and the numerous references therein. Elliptic solutions for the CNLS and Manakov system were derived in [22, 23, 24].

In the presence of external elliptic potential explicit stationary solutions for NLS were derived in [16, 25, 26]. These results were generalized to the n -component CNLS in [19]. For two-component CNLS explicit stationary solutions are derived in [27].

In this paper we study two multicomponent nonlinear systems for two boson and N_f fermion fields. They include as particular cases both the Manakov system [20] with external potential and Salerno model [8] describing quantum degenerate mixtures of bosons and fermions trapped in an optical lattice.

We derive exact solutions in Jacobi elliptic functions for these equations and some particular cases. These results are natural extension of the ones in our recent paper [12].