Closed Hypersurfaces in Riemannian Manifolds Whose Second Fundamental Forms are of Constant Length

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Abstract. In [2], Ye had proved the existence of a family of constant mean curvature hypersurfaces in an $m+1$-dimensional Riemannian manifold (M^{m+1}, g), which concentrate at a point p_0 (which is required to be a nondegenerate critical point of the scalar curvature). Recently Mahmoudi [1] extended this result to the other curvatures (the r-th mean curvature for $1 \leq r \leq m$). In this paper, using a similar idea, we show that there exist compact hypersurfaces whose second fundamental form are of constant length.

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INTRODUCTION

In recent years many new examples of hypersurfaces with constant mean curvature in the space forms have been constructed. On the contrary, it was believed that to construct hypersurfaces with constant mean curvature in general spaces is very difficult. But Rugang Ye [2] had proved the following amazing theorem in 1991.

Theorem 1 ([2]). *Let (M^{n+1}, g) be a Riemannian manifold. Suppose that p_0 is a nodegenerate critical point of the scalar curvature of M. Then there exists $r_0 > 0$, such that for all $\rho \in (0, r_0)$, the geodesic sphere $S_\rho(p_0)$ may be perturbed to a constant mean curvature hypersurface S_ρ with $H = 1/\rho$.*

This theorem shows that there exist many closed hypersurfaces with constant mean curvature in almost all compact Riemannian manifolds.

Recently Fethi Mahmoudi [1] extended this result to the k-th mean curvature H_k for $1 \leq k \leq m$. Here H_k are defined as

$$H_k(S) = \sum_{i_1 < \cdots < i_k} \kappa_{i_1} \cdots \kappa_{i_k}$$

where κ_j are the principal curvatures of the hypersurface S in M^{n+1}.

Theorem 2 (Mahmoudi [1]). *Let (M^{n+1}, g) be a Riemannian manifold. Suppose that p_0 is a nodegenerate critical point of the scalar curvature of M. Then there exists $r_0 > 0$, such that for all $\rho \in (0, r_0)$, the geodesic sphere $S_\rho(p_0)$ may be perturbed to a constant k-curvature hypersurface S_ρ with $H_k = C_n^k \rho^{-k}$.*

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