Travelling Waves for Some Generalized Boussinesq Type Equations

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Abstract. This paper deals with traveling wave solutions of the $(N+1)$-dimensional Boussinesq type equation and of the $B(m,n)$ equation. They are found into explicit form being expressed in some special cases by the Jacobi elliptic function. In the general case they are written into integral form and it turns out that they develop cusp type singularities.

Keywords: travelling wave solution, Boussinesq type equation, cusp type singularity, classical and generalized solution
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INTRODUCTION

1. In the recent several years different methods have been developed for establishing of exact special solutions of higher order Partial differential equations (PDE) describing nonlinear wave phenomena that appear in fluid mechanics, plasma physics, biology, hydrodynamics, etc. Concerning the well-known Boussinesq dispersive equation [1] and some possible generalizations (for example the $B(m,n)$ Boussinesq equation) they describe the formation of patterns in liquid drops, the vibrations of a single one-dimensional dense lattice and other effects. Some historical notes and facts on the subject could be found in the monographs [5, 3, 6], etc.

The classical approach for studying of conservative systems in mechanics with degree of freedom equal to one is very well described in [4] and by applying it or its modifications the traveling wave solutions of many equations of Mathematical Physics are found in [12] (see for example Chapter III).

2. This paper deals with the $(N+1)$ dimensional dispersive Boussinesq equation (including the shallow water waves equation obtained for $n=3$, $N=2$)

$$u_{tt} = u_{xx} + \lambda(u^n)_{xx} + \mu(u^m)_{xxxx} + \sum_{j=1}^{N-1} u_{y_j y_j} \tag{1}$$

where $N \geq 2$, $\lambda =$const, $\mu =$const, $\lambda\mu \neq 0$, $n \geq 2$, $m \geq 1$, $m,n \in \mathbb{N}$, $u = u(t,x,y_1,\ldots,y_{N-1})$, $(t,x) \in \mathbb{R}^2$ and $B(m,n)$ equation

$$u_{tt} = u_{xx} + \lambda(u^n)_{xx} + \mu(u^m)_{xxxx} \tag{2}$$

where $u = u(t,x)$, $(t,x) \in \mathbb{R}^2$, $m > 1$, $n > 1$, $m,n \in \mathbb{N}$, and the constants λ and μ satisfy $\lambda\mu \neq 0$.

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