FROM THE POINCARÉ–CARTAN FORM TO A GERSTENHABER ALGEBRA OF POISSON BRACKETS IN FIELD THEORY

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Abstract

We consider the generalization of the basic structures of classical analytical mechanics to field theory within the framework of the De Donder-Weyl (DW) covariant canonical theory. We start from the Poincaré-Cartan form and construct the analogue of the symplectic form – the polysymplectic form of degree $(n+1)$, n is the dimension of the space-time. The dynamical variables are represented by differential forms and the polysymplectic form leads to a natural definition of the Poisson brackets on forms. The Poisson brackets equip the exterior algebra of dynamical variables with the structure of a "higher-order" Gerstenhaber algebra. We also briefly discuss a possible approach to field quantization which proceeds from the DW Hamiltonian formalism and the Poisson brackets of forms.

1. INTRODUCTION

In this communication I discuss the canonical structure underlying the so-called De Donder–Weyl (DW) Hamiltonian formulation in field theory and its possible application to a quantization of fields. The abovementioned structure was found in a recent paper of mine,[1] to which I refer both for further references and for additional details. In particular, I am going to show that the relationships between the Poincaré-Cartan form, the symplectic structure and the Poisson structure, which are well known in the mathematical formalism of classical mechanics, have their natural counterparts also in field theory within the framework of the DW canonical theory. This leads to the analogue of the symplectic structure, which I call polysymplectic, and to the analogue of the Poisson brackets which are defined on differential forms.

Recall that the Euler-Lagrange field equations may be written in the following form (see for instance Refs. 2-4)

$$\frac{\partial p^i_a}{\partial x^i} = -\frac{\partial H}{\partial y^a}, \qquad \frac{\partial y^a}{\partial x^i} = \frac{\partial H}{\partial p^i_a} \tag{1.1}$$

in terms of the variables

$$p^i_a := \frac{\partial L}{\partial(\partial_i y^a)}, \tag{1.2}$$

$$H := p^i_a \partial_i y^a - L \tag{1.3}$$