SPECTRUM OF THE DIRAC OPERATOR ON THE SU(2) MANIFOLD AS ENERGY SPECTRUM FOR THE POLYANILINE MACROMOLECULE

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Abstract

A link is shown between the purely mathematical consideration of spinor structures on a group manifold and the description of a physical system. The $SU(2)$ manifold is taken as an example. The considered model describes physical systems with an order parameter taking values in the $SU(2)$ group, and predicts energy levels for fermions that satisfy the Dirac equation in this kind of system. The assumptions of the theoretical model are shown to be being fulfilled in the case of protonated polyaniline. The theoretical prediction of the polaron energy levels is compared with experimental data, showing a very good agreement.

1. INTRODUCTION

It has been known for a long time that geometric methods are very effective in some areas of physics. The classical example is the geometric description of gravity. Moreover the geometry of multidimensional space-time seems to be the proper framework for the unified theory of fundamental interactions, from simple Kaluza-Klein theory to supergravity and superstrings.[1] Therefore it is not surprising that the geometric methods from the pure field theory penetrate other areas of physics, in particular the theory of condensed matter.[2−5]

This work can serve as one example showing this influence. In the first, purely geometric, part of the work, we consider the spinor structures and spinor bundles (associated by the Dirac representation) on group manifolds. The example for which this construction is realized is the SU(2) manifold with the natural bi-invariant metric. This method gives also the spectrum of the Dirac operator on the considered manifold. The purpose of the next part of the work was to find experimental data corresponding to a physical system which could be described this way. We are looking for an application illustrating this geometrical construction and for checking its validity. The example has been sought in the area of the theory of condensed matter, because in this area the experimental data are usually more complete than in the traditional areas described by field theory, i.e. in cosmology or the theory of fundamental interactions. In fact in the physics of condensed matter the experimental verification of the theoretical predictions is also possible. In addition this area is not as deeply penetrated by the geometrical