ON THE DEFORMATION OF COMMUTATION RELATIONS *

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Abstract

Deformed commutation relations and corresponding consistency conditions with braid relations are studied in terms of the so-called Wick algebras. We discuss the construction of such algebras and give some examples.

1. INTRODUCTION

Recently the q-deformed commutation relations for creation and annihilation operators (CAO) have been studied from different points of views by several authors, see for example Refs. 1-6. The commutation relations for Hecke braiding have been studied by Kempf.[7] The deformed commutation relations have been also studied by Bożejko and Speicher.[8,9] It is interesting from the algebraic point of view that all deformations of the commutation relations for CAO can be described in terms of the so-called Wick algebras.[10] Note that Wick algebras are some special examples of R-Weyl algebras[11] (see also Refs. 12,13). In a Wick algebra, there are no relations between creation (or annihilation) operators themselves, but such relations are possible in certain cases.[10] Obviously all such relations should be consistent. Hence we need some additional assumption.[10] In this paper we are going to continue the study of deformed commutation relations and the corresponding consistency conditions in terms of Wick algebras. Our study of deformed commutation relations is based on two operators R and R'. These operators are not arbitrary, they must satisfy some consistency conditions like Wess-Zumino conditions for differential calculus on a quantum plane.[14] Our study is a continuation of previous papers.[15-18]

2. WICK ALGEBRAS

We are going here to develop the concept of Wick algebras for our study of deformed commutation relations. Note that our notion of such algebras is based on the paper of Jorgensen, Schmith, and Werner.[10]

DEFINITION A. *A Wick algebra is an algebra $\mathcal{W}(A)$ generated by two sets of generators x_i and x_i^*, for $i \in I$ i.e. $\mathcal{W}(A) = \mathbf{C} < x_i^*, x_i, i \in I >$ such that*
(i) $*$ is an involution

$$x_i^{**} = x_i,\ (x_i x_j)^* = x_j^* x_i^*,$$

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