THE q-DEFORMED QUANTUM MECHANICS IN THE COHERENT STATES MAP APPROACH

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Abstract

We study q-quantum mechanics in one degree of freedom. Among other things, we discuss the holomorphic representation of the q-deformed Heisenberg-Weyl algebra and its realization by covariant Berezin symbols.

1. COHERENT STATES MAP AND q-CANONICAL COMMUTATION RELATIONS

Let $\mathbf{D}_q$ be the open disc in $\mathbb{C}$ of radius $(1-q)^{-1/2}$ centered at zero, where $-1 < q < 1$, and let $\mathcal{M}$ be a complex separable Hilbert space with orthonormal basis $\{e_n\}$,$n = 0, 1, 2, \ldots$. We define the analytic map (the coherent states map),[1] $K_q : \mathbf{D}_q \to \mathcal{M}$ by

$$K_q(z) = \sum_{n=0}^{\infty} \left[\frac{(1-q)^n}{(1-q)\cdots(1-q^n)}\right]^{\frac{1}{2}} z^n e_n , \tag{1.1}$$

where for $n = 0$ one assumes that the coefficient in front of e_0 is equal to one. Since

$$\langle K_q(z)|K_q(z)\rangle = \sum_{n=0}^{\infty} \frac{(1-q)^n}{(1-q)\cdots(1-q^n)}(\bar{z}z)^n =: \exp_q(\bar{z}z) < \infty , \tag{1.2}$$

for $\bar{z}z < (1-q)^{-1}$, the definition is correct. The function $\exp_q$ is called[6] the q-deformation of the exponential function $\exp = \exp_1$. So for $q = 1$ one obtains the standard coherent states map (the Bargmann-Fock coherent states map) and for $q = 0$ one gets the geometric sequence, $\exp_0 \bar{z}z = (1-\bar{z}z)^{-1}$.

Having introduced the coherent states map, one can define an annihilation operator by

$$AK_q(z) := zK_q(z), \qquad \forall z \in \mathbf{D}_q. \tag{1.3}$$

This is a bounded operator, with norm $||A|| = (1-q)^{-1/2}$ for $0 \leq q < 1$ and $||A|| = 1$ for $-1 < q < 0$. The operators so defined satisfy q-Heisenberg commutation relations

$$[A, A^\dagger]_q := AA^\dagger - qA^\dagger A = 1 \tag{1.4}$$

which were studied in Refs. 3-5 and 9-12, for instance.