THE QUANTUM $SU(2,2)$-HARMONIC OSCILLATOR

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Abstract

We consider the classical $SU(2,2)$-harmonic oscillator, with phase space $\mathcal{A}(2,2) = SU(2,2)/S(U(2) \times U(2))$, and quantize it by the coherent state method. The quantum Hamiltonian is the Toeplitz operator corresponding to the square of the distance in the $SU(2,2)$-invariant Kähler metric on the phase space. Its spectrum is computed, and found to depend on the choice of representation of $SU(2,2)$.

1. INTRODUCTION

The $SU(2,2)$-harmonic oscillator is the generalization of the model harmonic oscillator on a flat phase space. In our case the phase space $\mathcal{A}(2,2) = SU(2,2)/S(U(2) \times U(2)) \simeq SO(4,2)/SO(4) \times SO(2)$ is the eight dimensional conformal domain, on which canonical coordinates $(x^\mu, p^\mu), \mu = 0, ..., 3$ can be globally introduced.

Spaces of this type are well known as Cartan classical domains. They appear in physics and mathematics and have been considered by many authors. The complex geometry of these spaces and, in particular, its applications in conformal theories has been investigated in the work of Coquereaux and Jadczyk (see Ref.1 and references therein). As the phase space of scalar massive conformal particle it has been considered by A. Odzijewicz.[2,3]

The geometry of $\mathcal{A}(2,2)$ is related to the space-time geometry. The Shilov boundary of $\mathcal{A}(2,2)$ is the compactified Minkowski space-time, endowed with the conformal structure of signature $(+,-,-,-)$. The compactification is obtained by adding a light cone at infinity to the usual Minkowski space-time.

As it is suggested in Ref.4, the conformal domain can be considered as a replacement of space-time on the microscale. This interpretation is based on Born's idea of a reciprocity symmetry between space-time and energy-momentum space. The reciprocity symmetry can be reformulated as the symmetry of the conformal domain. As a consequence, these spaces are not distinguished on the microscale. Minkowski space is interpreted as the very-high-mass, or very-high-energy-momentum-transfer limit of the conformal domain.

The $SU(2,2)$-harmonic oscillator is a one-body system. It is obtained from a two-body interacting system by introducing "center of the mass" coordinates. The interaction is $SU(2,2)$-invariant. The covariant harmonic oscillators are used in quark