GEOMETRO-STOCHASTIC QUANTIZATION AND QUANTUM GEOMETRY

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Abstract

The most basic features of the geometro-stochastic method of quantization are outlined in the nonrelativistic and special relativistic regime. Their adaptation to the general relativistic regime leads to the replacement of the classical frame bundles, which underlie the formulation of parallel transport in classical general relativity, with quantum frame bundles. This gives rise to quantum geometries for quantum field theory in curved spacetime, in which quantum frames take over the role played by complete sets of observables in conventional quantum theory. The ensuing quantum-geometric mode of propagation in general relativistic quantum bundles is implemented by path integration methods based on parallel transport along broken paths consisting of arcs of geodesics of the Levi-Civita connection, and can be further extrapolated to a geometric formulation of quantum gravity.

1. INTRODUCTION

The method of geometro-stochastic quantization was developed[1−4] originally in order to deal with still unresolved problems[5,6] in relativistic quantum particle and field localization. It eventually led,[7,8] however, to a geometric framework for quantum general relativity, capable of resolving foundational problems in quantum field theory (QFT) in curved spacetime and in quantum gravity.[9,10] In its use of phase space representations of the Galilei and the Poincaré group,[11] the geometro-stochastic method of quantization shares[12] some features with the well-known geometric method of Kostant[13] and Souriau;[14] whereas, in its use of coherent states, it displays common features (see Ref. 15, Sec. 16.2) with the method of Berezin.[16] However, in the ultimate analysis,[9,10] it transcends both these methods, since it proves applicable to the general relativistic regime. Indeed, at the physical level, geometro-stochastic quantization leads to a concept of quantum frame capable of taking over in the general relativistic regime the role played in conventional quantum theory by complete sets of observables. In QFT in curved spacetime, this mediates the formulation of quantum geometries that are Hilbert or pseudo-Hilbert fibre bundles associated with Poincaré frame bundles over curved spacetime. In turn, this enables a purely geometric formulation[17] of propagation of quantum fields in such bundles by means of path integrals that conform to the strong equivalence principle of general relativity. This kind of propagation is based on quantum