THE BERRY PHASE AND THE GEOMETRY OF COSET SPACES

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Abstract

We consider the Berry phase for quantum systems with dynamical symmetry and nondegenerate spectrum. It is shown that the corresponding phase factor is the holonomy group element of the group fibre bundle $G(G/S)$ where G is the dynamical symmetry group and S is its maximal commutative subgroup. The connection between such elements and Cartan structure 1-forms of the bundle is established. The particular case of $SU(3)$ symmetry group is completely investigated.

1. DYNAMICAL SYMMETRY AND GEOMETRIC PHASE

Physical effects connected with the geometric phase that appears in parametric quantum systems with quasi-periodic wave functions are now a subject of intensive investigation.[1] Such an effect can be mathematically expressed by a specific phase factor being acquired by the system's wave function when the corresponding parameters evolve periodically and there are no transitions in the system. This kind of phase factors was discovered by Berry[2] in the case of the evolution of adiabatic parameters and a nondegenerate spectrum. Concretely, let us consider a quantum system with Hamiltonian $H(\vec{R})$, where $\vec{R}$ denotes the set of parameters $\{R_i; i = 1, ..., N\}$ and is formally considered as a vector in $\mathbf{R^n}$. Let now $\vec{R}$ evolve periodically and classically, i.e. slowly enough for the adiabatic theorem[3] to be valid:

$$H(\vec{R})|n(\vec{R})>= E_n(\vec{R})|n(\vec{R})> \quad . \tag{1.1}$$

The equation (1.1) is quasi-stationary because $\vec{R} = \vec{R}(t)$ is a function of time. Comparing (1.1) with the non-stationary Schrödinger equation

$$i\frac{\partial}{\partial t}|\Psi(t)>= H(\vec{R}(t))|\Psi(t)>, \tag{1.2}$$

one can obtain the solution of the problem for $\vec{R}(0) = \vec{R}(T)$ at some T

$$|\Psi_n(T)>= exp(-i\Phi_n(T) + i\gamma_n(C))\ |n(\vec{R}(0))>, \tag{1.3}$$