SYMPLECTIC INDUCTION, UNITARY INDUCTION AND BRST THEORY (Summary)

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The data needed for a unitary representation of a Lie group G are a Hilbert space $\mathcal{H}$ (finite or infinite dimensional) and a (linear) action of G on $\mathcal{H}$. This action should preserve the inner product in $\mathcal{H}$. In the same spirit one can define a symplectic and a hamiltonian representation of G. The data needed are a symplectic manifold (M, ω) and an action of G on M. For a symplectic representation this action should preserve ω; for a hamiltonian representation it should also posses an equivariant moment map (which takes values in the dual Lie algebra of G). In these terms the procedure of geometric quantization can be described as a procedure to transform a hamiltonian representation of G into a unitary representation.

Now let $H \subset G$ be a closed subgroup of G. If we have a unitary representation of H, the well known procedure of unitary induction constructs out of this a unitary representation of G. If on the other hand we have a hamiltonian representation of H, the less well known procedure of symplectic induction constructs out of this a hamiltonian representation of G. It is thus quite natural to ask whether geometric quantization intertwines these two induction procedures.

We show that it is very probable that this is indeed the case. Although the general question is still open, in various special cases this has been proved. Moreover, no counter example is known. The link between this question and BRST theory is as follows. In the context of BRST theory it was found that for non-unimodular groups the standard Dirac procedure for the form of quantum constraints needs to be modified by a term involving the trace of the adjoint representation (which is zero for unimodular groups). Since constraints also appear in the symplectic induction procedure (in the form of a Marsden-Weinstein reduction), this modification also appears there. It then turns out that this modification is crucial to prove that geometric quantization intertwines the two types of induction procedures.

References

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