THE LORENTZ FORCE EQUATION IN TWISTOR TERMS - A SYMPLECTIC FRAMEWORK -

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ABSTRACT

Using Lorentz force equation as an input a Hamiltonian mechanics on the non-projective two twistor phase space TxT is formulated. Such a construction automatically reproduces dynamics of the intrinsic classical relativistic spin. The charge appear as a dynamical variable.

1. Introduction

The classical motion of a relativistic electrically charged massive and spinning particle exposed to an external electromagnetic field is, in Minkowski space, described by the Lorentz-Dirac (LD) force equation and by the so called Bargmann, Michel, Telegdi (BMT) equation for the intrinsic angular momentum (the spin).

If we denote by, X^a, P_a, S_a, F_{ab}, $m^2 := P^b P_b$, e and g, the four-position, the four-momentum, the Pauli-Lubański four-vector, the external electromagnetic field tensor, the mass squared, the charge and the gyromagnetic ratio of the particle then these Poincaré covariant equations may be written as follows:

$$\dot{X}^a = P^a, \tag{1.1}$$

$$\dot{P}_a = eF_{ab}P^b + D_a, \tag{1.2}$$

$$\dot{S}_a = \frac{ge}{2}F_{ab}S^b + \frac{ge}{2m^2}(F_{ik}S^iP^k)P^a - \frac{1}{m^2}(\dot{P}_kS^k)P^a \tag{1.3}$$

where

$$P_aS^a = 0 \tag{1.4}$$

and

$$D_aP^a = 0. \tag{1.5}$$

D_a is a small space-like correction four-vector (small compared with the space-like four-vector $eF_{ab}P^b$) containing higher derivatives of the external electromagnetic field F_{kl}, F^*_{kl} and terms nonlinear in the spin variable S^i [1,2]. When the particle forms (a classical limit of) an electron and the radiation damping effects are neglected the value of g equals 2 (the Dirac value).