QUANTUM PARTICLE ON A TORUS WITH AN EXTERNAL FIELD

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ABSTRACT

Quantization of a particle on a 2-torus T^2 is discussed within the framework of quantum Borel kinematics and applied and generalized to a 'charged' particle coupled to an inhomogeneous external 'magnetic' field modelled via a closed 2-form on T^2.

1. Introduction

The study of quantum systems in two dimensions is important in condensed matter physics as well as in quantum field theory and statistical mechanics (2D-structures, quantized Hall effect, anyons [1]).

A convincing method to quantize a system localized on a (smooth, finite dimensional) manifold M is the *Borel quantization* [2],[3] sketched in section 2, based on the geometry of a complex line bundle over M. We discuss the example of a 2-dimensional Torus $T^2 = S^1 \times S^1$ in section 3 and we generalize in section 4 the Borel quantization scheme such that it is possible to interprete a closed 2-form on T^2 as an external field F or a topological potential of the 3rd kind [5] interacting with the particle.

There are alternative approaches like the group-theoretic quantization [6], which was also applied to a particle on T^2 coupled to a *homogeneous* external magnetic field [7]. We mention that, in contrast to such *ad hoc* group-theoretical constructions, our method leads to results which show the direct relation between quantization on the one hand and relevant *topological* properties of T^2 on the other. Furthermore, we treat also *inhomogeneous* fields which break the symmetry on which the quantization of [7] is based. In section 5 we comment on the Borel quantization for a particle localized on an arbitrary 2-dimensional closed orientable surface in the presence of