SYMPLECTIC AND KÄHLER COHERENT STATE REPRESENTATIONS OF UNIMODULAR LIE GROUPS

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ABSTRACT

An outline is given of a classification theory for unimodular Lie groups that possess Kähler coherent state representations, i.e. irreducible unitary representations admitting a complex orbit on the projective space corresponding to the representation space.

Introduction

A system of coherent states in the sense of Perelomov [1] for a unitary repre-
entation of a Lie group G is a G-orbit on the projective space of all lines in the
epresentation space. This definition is quite general, so usually one has to impose
ome conditions on the coherent state orbit. From physical point of view, the most
ppealing is the requirement that the orbit be a symplectic manifold (with a sym-
lectic structure induced by the imaginary part of the Fubini-Study metric on the
rojective space), since then the orbit may be interpreted as the classical phase
pace of a mechanical system with symmetry group G embedded into the quantum
hase space. Such an embedding is the starting point of a quantization theory
ecently proposed by Odzijewicz. [2,3] A still more restrictive condition is that the
oherent state orbit be complex, and hence Kählerian. This case plays an important
ole in Berezin's quantization. [4,5,6]

Of course, not every representation admits symplectic and, all the more,
ähler coherent state orbits, so it is an interesting problem to classify such rep-
esentations; in particular, to classify groups possessing such representations. For
ome special classes of groups the solution to this problem is well known, but the
eneral case seems to be unsolved.

In this paper we present the main ideas of the proof of such a classifica-
ion theorem for the case of unimodular Lie groups. As compared to our earlier
nnouncement, [7] this paper is more detailed an hence, we hope so, more compre-
ensible. A detailed presentation of these results, with complete proofs, is still in
reparation and will appear elsewhere. For pedagogical reasons we survey also some
ell known results.