GROUP REPRESENTATIONS AND QUANTIZATION OF THE MOMENTUM MAP

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ABSTRACT

In this paper we treat a general method of quantization of Hamiltonian systems whose flow is a subgroup (not necessarily closed) of a torus acting freely and symplectically on the phase space. The quantization of some classes of completely integrable systems as well as the Borel-Weil-Bott version of representation theory are special cases.

Recently there has been great interest in the quantization of compact phase spaces [1-4]. These spaces do not have in general globally defined potentials of their symplectic forms. This is a serious obstacle for the application of the Schrodinger quantization scheme the issue is settled by the geometric quantization of Kostant [5] and Souriau [6]. Quantization of classical phase spaces is also a general method for obtaining representations of semisimple Lie groups [5]. The procedure is straightforward, but the explicit description of a given representation requires explicit coordinates on the phase space. Obviously, the compact phase spaces can not be parametrized globally. The way to surround the computations with local coordinates is proposed by Gates et al [2]. These authors show how to treat phase spaces of the form $\mathbb{G}/\mathbb{T}$ where $\mathbb{G}$ is a compact Lie group and $\mathbb{T}$ is a maximal torus subgroup. Spaces of this type are known as flag manifolds. The key idea in [2] is that instead of direct quantization of relevant coset spaces one can consider larger flat spaces with Poisson bracket structures, and then following Dirac to quantize the constrained systems. The results obtained in [2] look so nice and the technique that has been used is so simple to deserve as such some comments. Especially one can ask the natural question about the place and role of geometric quantization in this situation. One of the purposes of this paper is by tracing back the principal stages of the mentioned construction to compare it with geometric quantization.

Let $X \to M$ be a principal fibre bundle, where the global space X is a compact smooth manifold, the structure group $\mathbb{T} = U(1)^{\times k}$ is a k-dimensional torus, and the base M is a symplectic manifold.