COHERRENT STATES FOR REDUCED PHASE SPACES

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ABSTRACT

In this paper we show how to construct coherent states for a reduced physical system if we know such states for the system before reduction. Applications of this construction to concrete physical situations are also presented.

1. Introduction

In [3] it was shown that any physical system can be described by the followi triple of geometrical objects:

i) a finite-dimensional manifold M;
ii) a complex Hilbert space $\mathcal{M}$;
iii) a map $\mathcal{K}: M \to \mathbf{CP}(\mathcal{M})$ (coherent states map).

The mechanical systems which we want to discuss here are distinguished symplecticity of the map $\mathcal{K}: M \to \mathbf{CP}(\mathcal{M})$, i.e. the requirement that the pull-ba ω_{FS} of the Fubini-Study form ω be a symplectic form. According to [3], the ener function $h: M \to \mathbf{R}$ of the mechanical system is related to $(M, \mathcal{M}, \mathcal{K}: M \to \mathbf{CP}(\mathcal{M}$ by an integral equation. Therefore, having the coherent states map, we can descri the system completely. Among others things, we can find the family of observabl characteristic for the system, and its Lagrangian. An exhaustive description of t approach to mechanics is given in [3].

In this paper (see §2) beginning with $(M, \mathcal{M}, \mathcal{K}: M \to \mathbf{CP}(\mathcal{M}))$, we constru the coherent states map $\tilde{\mathcal{K}}: \tilde{M} \to \mathbf{CP}(\tilde{\mathcal{M}})$ for the reduced phase space $\tilde{M}$. T quotient manifold $\tilde{M}$ is obtained from M by the reduction procedure (see [1 using the system of commutating observables $f_1, \ldots, f_n \in \mathcal{C}^\infty(M, \mathbf{R})$, $\{f_i, f_j\} =$ which admit the Ehrenfest quantization (see [3]). The reduced coherent states m $\tilde{\mathcal{K}}: \tilde{M} \to \mathbf{CP}(\tilde{\mathcal{M}})$ and the reduced projective Hilbert space $\mathbf{CP}(\tilde{\mathcal{M}})$ are defin in the natural way, i.e. by the quantum reduction of the coherent states $\mathcal{K}(m$ $m \in M$. The usefulness of the reduction procedure proposed in §2 is confirmed §3 by its application to concrete physical systems. Let us also note that consideri the mechanical system as a triple $(M, \mathcal{M}, \mathcal{K}: M \to \mathbf{CP}(\mathcal{M}))$, one finds a natu relation between the classical and quantum reduction procedure.