QUANTUM MECHANICS ON Z_M AND q–DEFORMED HEISENBERG–WEYL ALGEBRAS

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Abstract

The finite–dimensional quantum mechanics yields a more convenient operator basis for representation of q–deformed Heisenberg-Weyl (q–HW) algebras when q is a root of unity, i.e. $q^M = 1$. Two free parameters appear when the representation is constructed. Moreover, the irreducibility of the representations is discussed.

1. INTRODUCTION

In Refs. 1,2, the authors propose a method to construct so called q–boson realizations of quantum algebras from their Verma representations. The method was illustrated on examples q-HW algebras, $U_q(sl(2,\mathrm{C}))$, $U_q(sl(3,\mathrm{C}))$ and $U_q(sl(n+1,\mathrm{C}))$. The finite–dimensional quantum mechanics on Z_M provides a more natural basis for representations of these algebras if $q = e^{\frac{2\pi i}{M}}$, $M = 2,3,\cdots$. Starting from the basic relations of quantum mechanics on discrete finite space in Section 2, a family of representations is constructed in Section 3 and in Section 4 their irreducibility is discussed. The q-HW algebras are defined as associative algebras W_2^q over C generated by b^+, b and $q^{\pm N}$ satisfying [3]

$$q^N q^{-N} = q^{-N} q^N = 1, \tag{1.1}$$

$$q^N b^{\pm} q^{-N} = q^{\pm 1} b^{\pm}, \tag{1.2}$$

$$bb^+ - q^{\mp 1} b^+ b = q^{\pm N}, \quad (b^- = b), \tag{1.3}$$

which degenerates to the usual HW algebras in the limit $q \to 1$.

2. FINITE–DIMENSIONAL QUANTUM MECHANICS ON THE CYCLIC GROUP Z_M

Formulations of the finite–dimensional quantum mechanics (FDQM) has been made in several papers.[4,5,6] Following Ref. 4 we shall present the basic relations of FDQM. For the sake of simplicity we shall restrict our attention to one classical degree of freedom. Theories for more degrees of freedom can be obtained as a tensor product of theories of one degree.

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