SYMPLECTIC REALIZATIONS OF THE GALILEI-CARROLL GROUP

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Abstract

We generalize the Galilei-Carroll group G,[1,2] we classify all his G-elementary systems under the coadjoint action of G on the dual $\mathcal{H}^*$ of the Lie algebra $\mathcal{H}$, the central extension of the Lie algebra of G.

1. THE GALILEI-CARROLL GROUP

Let M be a $(n+1)$-dimensional space endowed with the metric

$$\eta_{\mu\nu} = \mathrm{diag}(0, -1, ..., -1). \tag{1}$$

Let $x^\mu = \begin{pmatrix} \xi \\ x^i \end{pmatrix}$, $i = 1, \dots, n$, be the coordinates of an arbitrary point of M. Then the displacement group associated with (1) is the group of transformations

$$x_2{}^\mu = g_\nu{}^\mu \; x_1{}^\nu + x^\mu, \tag{2}$$

where

$$x^\mu = \begin{pmatrix} \xi \\ x^i \end{pmatrix}, \quad (g_\nu{}^\mu) = \begin{pmatrix} 1 & u_j R_i{}^j \\ 0 & R_i{}^j \end{pmatrix}, \quad (R_i{}^j) \in SO(n). \tag{3}$$

Under the usually composition law of matrices, one can verify that the group of transformations (2) is exactly the Carroll group[3], whose multiplication law is

$$\begin{aligned} &\left(\xi, x^i, u_i, R_k{}^i\right)\left(\xi', x'^k, u'_k, R'_j{}^k\right) \\ &\qquad = \left(\xi + u_i R_k{}^i x'^k + \xi', x^i + R_k{}^i x'^k, u_i + R_i{}^k u'_k, R_k{}^i R'_j{}^k\right). \end{aligned} \tag{4}$$

Now, let $V = M \times \mathbb{R}$ be a $(n+1)$-dimensional space, where $\mathbb{R}$ supports the absolute time coordinate. Let $x^a = \begin{pmatrix} x^\mu \\ \xi \end{pmatrix}$ be the coordinates of an element in V. Then one verifies that the isotropy and homogeneity of V together with the galilean principle of relativity of motion,[2] the isotropy and homogeneity of time, admit only the transformations

$$\begin{aligned} x_2{}^\mu &= g_\nu{}^\mu \; x_1{}^\nu + v^\mu t_1 + x^\mu \\ t_2 &= t_1 + t, \end{aligned} \tag{5}$$

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