# COVARIANT AND CONTRAVARIANT BEREZIN SYMBOLS OF BOUNDED OPERATORS

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**Abstract**

The algebras of covariant and contravariant Berezin symbols for bounded operators are described and a criterion is given for obtaining the symbol of a Hilbert-Schmidt operator. It is also proved that the operators $P_1 : L^p(M, d\mu) \to \mathcal{LT}_p(\mathcal{M})$ and $\langle\cdot\rangle : \mathcal{LT}_p(\mathcal{M}) \to L^p(M, d\mu)$, for $1 \leq p \leq \infty$, are continuous and linear and that the $C^*$-algebra of all bounded operators is obtained as the weak closure of $P_1(L^p(M, d\mu))$.

Following Ref.1, by a physical system we shall understand the following triple:

a) a finite dimensional differentiable manifold $M$;

b) a complex separable Hilbert space $\mathcal{M}$;

c) a differentiable map of manifolds, $\mathcal{K} : M \to \mathbf{CP}(\mathcal{M})$;

where $\mathbf{CP}(\mathcal{M})$ is the complex projective space of $\mathcal{M}$. The map $\mathcal{K}$ will be called the *coherent states map.* The states $\mathcal{K}(m)$, $m \in M$, by definition, will be the coherent states of the system.[2-4]

In the sequel, we shall always assume the existence of the resolution of the identity

$$\mathbf{1} = \int_M P(m) d\mu(m) \tag{1}$$

for some positive regular measure $\mu$ on the manifold $M$. Here $P(m)$ denotes the (orthogonal) projection operator corresponding to the coherent state $\mathcal{K}(m)$. Integration in (1) is meant in the weak sense.[5] This assumption is natural from the physical point of view because it implies the rule of composition of transition amplitudes between coherent states with respect to the measure $\mu$. From (1) it follows that the linear span of the coherent states forms a dense linear subspace in $\mathcal{M}$. Consider the universal (tautological) line bundle $\mathbf{E}:=\{(v,l) \in \mathcal{M}\times\mathbf{CP}(\mathcal{M}) : v \in l\}$ over the projective space $\mathbf{CP}(\mathcal{M})$. The bundle map $\pi : \mathbf{E} \to \mathbf{CP}(\mathcal{M})$ is a projection on the second component of the product $\mathcal{M}\times\mathbf{CP}(\mathcal{M})$. $\mathbf{E}$ is a holomorphic line bundle and has a canonically defined metric structure $\mathrm{H}^{\mathrm{FS}}$(via the scalar product $\langle\cdot|\cdot\rangle$ in the Hilbert space $\mathcal{M}$). The metric connection

$$\nabla^{\mathrm{FS}} : \Gamma^\infty(\mathbf{E}, \mathbf{CP}(\mathcal{M})) \to \Gamma^\infty(\mathbf{E}\otimes\mathrm{T}^*(\mathbf{CP}(\mathcal{M})), \mathbf{CP}(\mathcal{M}))$$

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