PLÜCKER EMBEDDING OF THE HILBERT SPACE GRASSMANNIAN AND BOSON-FERMION CORRESPONDENCE VIA COHERENT STATES

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Abstract

In this note we give a Plücker type description of the image of the embedding of the Hilbert space grassmannian of Segal and Wilson, obtained by resorting to the theory of quasi-free states of the CAR algebra. We also derive a boson-fermion correspondence via diastatic identities and coherent states.

1. INTRODUCTION

In this note we report on joint work[1,2] with Giorgio Valli. We employ the theory of quasi-free states of the CAR algebra[3–5] to obtain explicit equations describing the Plücker embedding of the Hilbert space grassmannian of Segal and Wilson (see e.g. Refs. 6,7), which is the basic object underlying loop group representation theory and conformal field theories.

Plücker type equations have already been formally written down for various explicit grassmannians coming from the KP hierarchy (see e.g. Ref. 8, and references therein). We adopt a C^*-algebraic point of view and show that the Plücker equations essentially amount to the Pauli Exclusion Principle.

We also reinterpret some diastatic identities of Ref. 1 in terms of a boson-fermion correspondence (§3). The latter fact is well known in many other different guises (see e.g. Refs. 6 and 8-16). The Kähler geometry of the Grassmannian appears to be neatly related to its CAR algebraic reinterpretation.

We have included, in §2, a short outline of the basic mathematical tools we employ, for the sake of readability. Complete proofs will be found in Ref. 2. This note should be seen as an abridged version of that work.

2. PRELIMINARIES

We begin by briefly reviewing the basic facts concerning the CAR algebra and the theory of quasi free-states. We refer to Refs. 3-5 for a complete treatment.

Given a complex, separable Hilbert space K (the *one-particle* space), the CAR (Canonical Anticommutation Relations) algebra $A(K)$ is the unital C^*-algebra generated by *creation* operators $a(f)^*$ (depending linearly on $f \in K$) and their adjoints

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