Some Recent Developments in Finding Systematically Conservation Laws and Nonlocal Symmetries for Partial Differential Equations

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Abstract This chapter presents recent developments in finding systematically conservation laws and nonlocal symmetries for partial differential equations. There is a review of local symmetries, including Lie's algorithm to find local symmetries in evolutionary form and their applications. The Direct Method for finding local conservation laws is reviewed and its relationship to and extension of Noether's theorem are discussed. Moreover, it is shown how symmetries, including discrete symmetries may yield additional conservation laws from known conservation laws. Systematic procedures are presented to seek nonlocally related PDE systems for a given PDE system with two independent variables. In particular, these procedures include the use of conservation laws, point symmetries, and subsystems (including subsystems arising after appropriate invertible transformations of variables) to obtain trees of equivalent nonlocally related PDE systems. In turn, it is shown how the calculation of point symmetries of such nonlocally related systems leads to the discovery of nonlocal symmetries for a given PDE system. The situation of systematically constructing useful nonlocally related systems in multidimensions is considered. Many illustrative examples are provided.

1 Introduction

This chapter is concerned with recent developments in finding conservation laws (CLs) and nonlocal symmetries for partial differential equations (PDEs). It focuses on recent research of the authors and some of the first author's collaborators, including Stephen Anco, Alexei Cheviakov, Temuer Chaolu, Jean-François Ganghoffer, Nataliya

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