Symbolic Computation of Nonlocal Symmetries and Nonlocal Conservation Laws of Partial Differential Equations Using the GeM Package for Maple

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Abstract The use of the symbolic software package `GeM` for `Maple` is illustrated with examples of computation of nonlocal symmetries and nonlocal conservation laws of nonlinear partial differential equations. In the considered examples, the nonlocal symmetries and conservation laws arise as local symmetries and conservation laws of potential systems. Full `Maple` code with detailed comments is presented. Examples of automated symmetry and conservation law classification are included.

1 Introduction

The majority of contemporary mathematical models involving partial and ordinary differential equations (PDE, ODE) are essentially nonlinear. The analysis of such models often proceeds using approximate, numerical, and/or problem-specific methods. In particular, the efficiency and precision of numerical solutions is commonly restricted by nonlinear effects, which limit mesh sizes and boost computation times, as well as by extra large data structures arising in discretizations of multi-dimensional problems.

Methods based on the framework of symmetry and conservation law analysis can be systematically applied to wide classes of PDE and ODE models. This research area, pioneered by Sophus Lie and Emmy Noether, has been recently developed in various directions, having become a set of interrelated methods that can provide essential analytical information about the underlying equations. For further details, an interested reader is referred to [6, 9, 10, 15, 26, 33].

For ODEs, seeking conservation laws is equivalent to seeking integrating factors; conserved quantities (first integrals) lead to the reduction of order. Conser-

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