Construction of Conservation Laws Using Symmetries

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Abstract The concept of *nonlinear self-adjointness* of differential equations, introduced by the author in 2010, is discussed in detail. All linear equations and systems are nonlinearly self-adjoint. Moreover, the class of nonlinearly self-adjoint equations includes all nonlinear equations and systems having at least one *local* conservation law. It follows, in particular, that the *integrable systems* possessing infinite set of *Lie-Bäcklund* symmetries (*higher-order tangent transformations*) are nonlinearly self-adjoint. An explicit formula for conserved vectors associated with symmetries is provided for all nonlinearly self-adjoint differential equations and systems. The number of equations contained in the systems under consideration can be different from the number of dependent variables. A utilization of conservation laws for constructing exact solutions is discussed and illustrated by computing non-invariant solutions of the Chaplygin equations in gas dynamics.

1 Nonlinear Self-Adjointness

The concept of self-adjointness of nonlinear equations was introduced [1, 2] for constructing conservation laws associated with symmetries of differential equations. To extend the possibilities of the new method for constructing conservation laws the notion of quasi self-adjointness was suggested in [3]. I introduce here the general concept of *nonlinear self-adjointness*. It embraces the previous notions of self-adjoint

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