

Symmetries and Some Special Solutions of the Helfrich Model

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Abstract The goal of this chapter is to present the results of the Lie group analysis in application to the Helfrich spontaneous curvature model. Special attention is paid to the translationally invariant solutions and the corresponding cylindrical equilibrium shapes. Graphs of closed diretrices of the obtained cylindrical surfaces in fixed and moving reference frame are presented.

1 Introduction

The *Helfrich model* (also known as the *spontaneous curvature model*) describes equilibrium shapes of *fluid membranes*—elastic membranous systems formed in aqueous solutions, such as biological membranes, soap bubbles, etc. [1, 2]. The governing equation of the Helfrich model is the *Helfrich equation*. In Mongé representation the Helfrich equation is a fourth order nonlinear partial differential equation. By using conformal coordinates and new dependent variables the Helfrich equation is transformed to a system of four nonlinear partial differential equations of second order of the derivatives [3, 4] (Sect. 2). We name this system the *Helfrich system*. Our main objective in this chapter is to present the results of the Lie symmetry group analysis in application to the Helfrich system of differential equations. By

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